RELOCATION OF LOCAL EARTHQUAKES BY SEISMIC RAY TRACING

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Abstract. Seismic ray tracing is used to relocate a set of local earthquakes recorded by a dense seismic array in Bear Valley, California. The crustal velocity structure is two-dimensional and incorporates most of the known and inferred velocity differences in the region. Relocated hypocenters fall within the San Andreas fault zone, and P residuals, without elevation or station corrections, now appear to be more internally consistent in the rms sense. Ray takeoff angles and azimuths are distinctly different from those of a uniformly layered earth model. Focal mechanism solutions using only traced data are well determined and have few inconsistent data. Zones of convergence and divergence of rays suggest large variations in observed amplitudes across the local network.

Introduction

With the accumulation of abundant data from the dense network of permanent and temporary seismographs established by the U.S. Geological Survey (USGS) in central California, it became clear that velocity differences across the San Andreas fault introduce a westward bias in hypocenters determined by using a layered crustal model [Brown and Lee, 1971; Lee et al., 1972a]. One way of accounting for the resulting misalignment of epicenters and fault trace is to use more than one velocity model in the hypocenter solution. For example, the location program HYPO71 [Lee and Lahr, 1972] was modified such that a different layered crustal model may be assumed under each station. By using three crustal models of one layer over a half space and a different station correction for each station it is possible to relocate Bear Valley earthquakes on the San Andreas fault with average residuals comparable to reading errors.

In this paper we shall test a method for locating local crustal earthquakes by ray tracing within a crustal velocity structure of arbitrary complexity. This method was first successfully applied by Engdahl [1973] to a plate model of the central Aleutians using both local and teleseismic arrival times from intermediate depth events. We shall extend this method to a set of earthquake data obtained by the U.S. Geological Survey's Bear Valley experiment and examine the kinds of effects produced by a laterally varying velocity structure.

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Seismic Ray Tracing

Recently, Eliseevnin [1965] and Julian [1970] have formulated the seismic ray theory applicable to the problem of ray tracing in a generally heterogeneous medium. Starting with the eikonal equation, Eliseevnin derived a system of six simultaneous differential equations in Cartesian coordinates for the motion of a disturbance along a ray. Julian [1970] used the calculus of variations and Fermat's principle of stationary time to derive a more seismologically useful form of these equations in spherical coordinates. Julian's solution in geocentric spherical polar coordinates is repeated below as a system of five simultaneous first-order differential equations giving the variation with time of the position (r, θ, ϕ) of a disturbance on a ray and the direction of propagation (i is incident angle, α is azimuth) from this disturbance in terms of the wave speed V and its spatial derivatives in the medium.

$$\dot{r} = -V \cos i$$

$$\dot{\theta} = -(V/r) \sin i \cos \alpha \qquad (1)$$

$$\dot{\phi} = [V/(r \sin \theta)] \sin i \sin \alpha$$

$$\dot{i} = -\sin i(\partial V/\partial r - V/r) + [(\cos i)/r][\cos \alpha(\partial V/\partial \theta) - (\sin \alpha/\sin \theta)\partial V/\partial \phi]$$

$$\dot{\alpha} = \sin i \sin \alpha[(V/r) \cot \theta] - [1/(r \sin i)][\sin \alpha(\partial V/\partial \theta) + (\cos \alpha/\sin \theta)\partial V/\partial \phi]$$

Julian has developed a computer program to treat this generalized formulation, and this program forms the basis for the calculations presented in this paper. Only the size of the computer core limits the detail to which V may be represented in the model. In application to the San Andreas fault it is convenient to represent the structure by a two-dimensional grid, normal to the fault, in which velocities are specified at discrete points. With this representation any degree of structural complexity may be defined, subject only to the grid interval. Cubic spline interpolation over a 16-point moving box within the velocity grid is used to determine continuous values of V and its first spatial derivatives as the ray propagates through the model. Numerical solution of the differential equations (1) is carried out by using the step size extrapolation

method of Bulirsch and Stoer [1966]. In practice, a maximum time step is chosen so that as the ray propagates with the time step the 16-point interpolation boxes along the ray overlap one another, and the ray must sense all velocity variations along its path. The grid size is chosen to be about the same order as the wavelength of the seismic waves used. In this paper we used a 0.5-s maximum step size and a velocity model discretized at 1-km depth intervals and 1-km surface distance intervals.

An iterative scheme introduced by Geiger [1910] forms the basis for most location programs now in use. Engdahl [1973] has modified this conventional iterative hypocenter location method for a spherically symmetric earth to utilize seismic ray tracing through a laterally varying structure. In Geiger's method we linearize the problem and solve by iteration. If $A = f(H, \Theta, \Phi, h)$, where A is the observed arrival time, H the origin time, Θ the epicenter co-latitude, Φ the epicenter longitude, and h the depth of focus, then expanding A about H_o, Θ_o, Φ_o, and h_o to first order and defining $A = H + T$, where T is the travel time, we obtain

$$A - A_o = \delta H + (\partial T/\partial \Theta)_o \, \delta\Theta + (\partial T/\partial \Phi)_o \, \delta\Phi + (\partial T/\partial h)_o \, \delta h \qquad (2)$$

where the subscript o indicates the value for the initial source parameters. Once the ray to a station from the initial source is found by ray tracing, T_o is known, and its derivatives at the source are determined explicitly from the relationships

$$\partial T/\partial \Theta = (r \sin i \cos \alpha)/V$$

$$\partial T/\partial \Phi = -(r \sin i \sin \Theta \sin \alpha)/V \qquad (3)$$

$$\partial T/\partial h = -(\cos i)/V$$

In practice, the problem of finding that particular ray that arrives at a given station is formidable and is done by iteratively shooting rays until one is found in the neighborhood of the desired end point. The wave front of that ray is then propagated the small distance remaining to determine T_o. The system of (2) is overdetermined, and least squares estimates of δ can be solved for iteratively.

Crustal Model and Earthquake Data

Several seismological studies have been made in the Bear Valley area of central California (about 40 km southeast of Hollister) because of relatively high seismicity and station density. For example, Wesson [1971] applied an iterative linear inversion method including ray tracing in a heterogeneous medium to travel time data obtained at a temporary seismic array from a single explosion located in Bickmore Canyon of the Bear Valley area. Ellsworth [1975] made a detailed study of the earthquake sequence of February-March 1972 near Bear Valley. Boore and Hill [1973] and Healy and Peake [1975] studied the seismic velocity structure along a section of the San Andreas fault near Bear Valley and noted the large velocity changes in the vicinity of the fault.

During the summer of 1974 a detailed monitoring of local earthquakes was carried out in the Bear Valley area by the U.S. Geological Survey under the leadership of J. H. Healy. The 100-

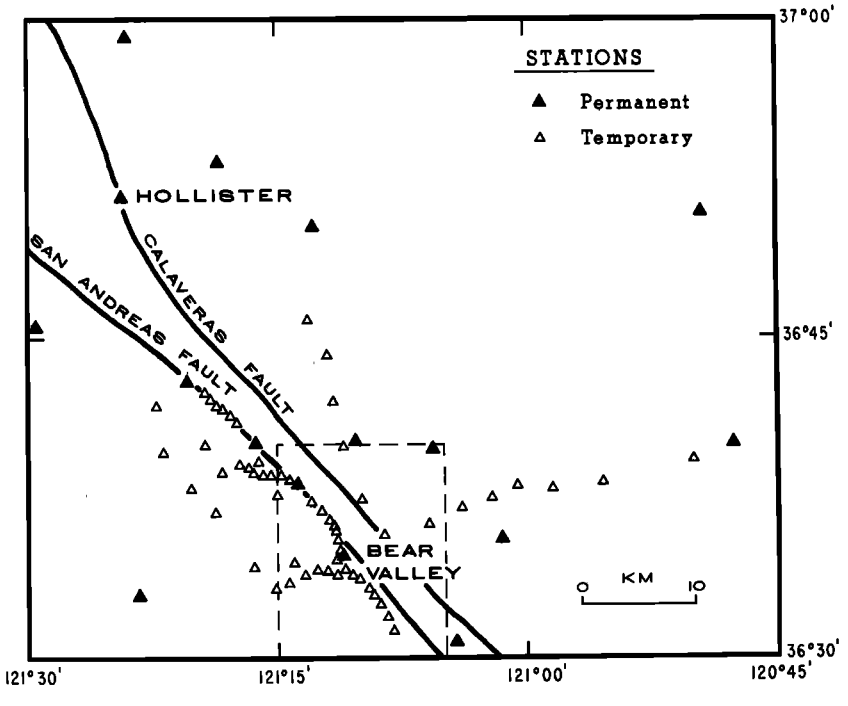

Fig. 1. Index map of area southeast of Hollister, California, showing the distribution of permanent and temporary seismic stations. Area enclosed by dashed lines indicates Bear Valley.

plus stations of the USGS Central California Microearthquake Network were augmented by 51 'centipede' stations [Fischer et al., 1975] and 14 independently recorded portable stations. Figure 1 shows the distribution of permanent and temporary stations in the general region southeast of Hollister; the area enclosed by the dashed line is taken here as the Bear Valley area.

A geologic cross section normal to the San Andreas fault was constructed by Roger Stewart (personal communication, 1975) based on the advice and geologic mapping of Dibblee [1975]. After considering the crustal structure determined by seismic refraction work [Stewart, 1968; also USGS unpublished data] as well as studies cited above, Stewart and Lee derived the crustal model shown in Figure 2a. The reference point on the fault zone is taken at 36°34.32'N and 121°09.89'W with the structure normal to a strike of the San Andreas fault of N42.9°W at this point. The structure is represented in the computer at 1-km intervals by a grid 18 km in depth by 178 km in surface distance perpendicular to the strike of the San Andreas fault. Because we specify the crustal structure model in 1-km grid intervals, the two grid lines at ±0.5 km from the surface trace of the San Andreas fault are accented in Figure 2a, although the actual fault zone extends 4 km further to the east. The velocities within the accented grid lines were assumed to be the average of the velocities at grid points on either side of the 1-km-wide 'San Andreas' zone.

Among the several hundred local earthquakes recorded during the USGS Bear Valley experiment, three well-recorded events of different focal depths were chosen for relocation by seismic ray tracing. The epicenter locations of these three events are shown in Figure 3, where the San Andreas fault zone is typically about 4 km in width, bounded by the surface trace of the Calaveras fault to the east and by the surface trace of the San Andreas fault to the west.

First P arrivals were read for the three selected events from all available stations. Seismic signals were recorded on 16-mm films for the permanent network stations. The films were projected with a 30X magnification so that 1.5 cm in length equals 1 s in time. Arrivals were picked manually by using a digitization table with reading errors of about ±0.03 s. Seismic signals for the centipede stations were recorded on 1-inch-wide magnetic tapes and were played back on paper so that 2.54 cm in length equals 1 s in time. Again, the arrivals were picked manually by using a digitization table with reading errors of about ±0.03 s. Seismic signals for the independently

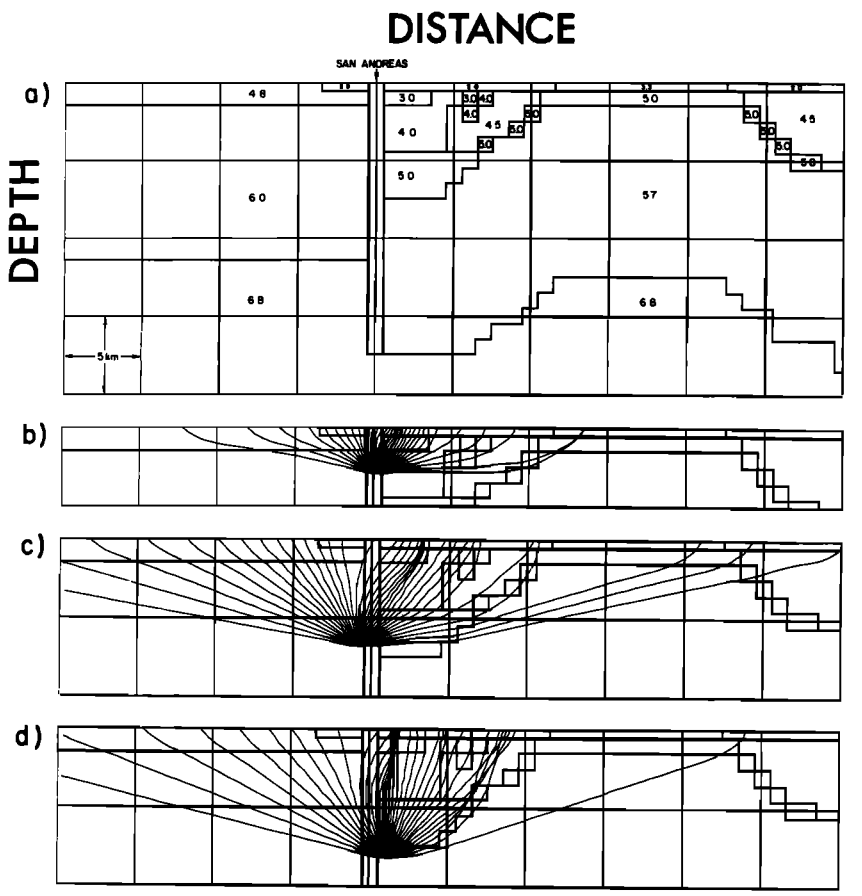

Fig. 2. Two-dimensional velocity model of Bear Valley area and rays normal to San Andreas fault traced at 5° increments of takeoff angle from relocated hypocenters. Velocities are in kilometers per second. (a) Velocity model (see text for explanation); (b) traced rays for event 1; (c) traced rays for event 2; (d) traced rays for event 3.

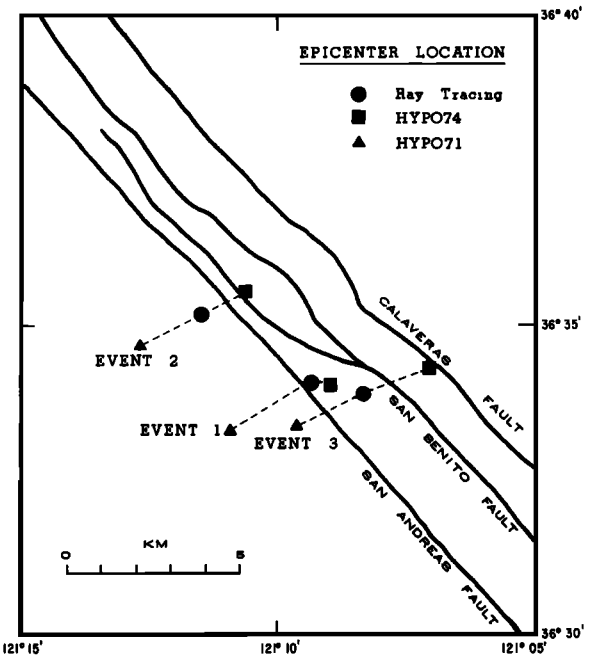

Fig. 3. Epicenter location for events 1, 2, and 3 by 'ray tracing', 'HYPO74', and 'HYPO71' procedures.

recorded portable stations were recorded on 0.5-inch-wide magnetic tapes and were played back on paper so that 1.27 cm in length equals 1 s in time. The arrivals were picked and timed manually with reading errors of about ±0.05 s.

Location Results

In the routine analysis of the Central California Microearthquake Network [Lee et al., 1972b] the location program HYPO71 [Lee and Lahr, 1972] is used with a uniformly layered model across the San Andreas fault. The large changes in the velocity and thickness of near surface rocks (known from explosion results) are modeled by selecting a layer of constant velocity (4.0 km/s) and varying its thickness for each seismic station according to whether the earthquake was east or west of the San Andreas fault. Since it is impossible to model the complex crustal structure of the large region covered by the network with such a simple model, systematic bias of a few kilometers in hypocenter locations is expected [Brown and Lee, 1971; Lee et al., 1972a]. Nevertheless, such a 'standard' location is routinely carried out for the sake of simplicity and continuity of network results.

In order to deal with the large lateral velocity variations across the San Andreas fault for the USGS Bear Valley experiment, the HYPO71 program was modified into 'HYPO74' by W. H. K. Lee so that a different layered model with station corrections could be assumed under each seismic station. The Bear Valley area was divided by W. L. Ellsworth into three regions, each region being modeled by a layer over a half space. Station corrections were included. Although this procedure succeeds in locating Bear Valley earthquakes near the San Andreas fault with rms errors of P residuals comparable to the reading errors, it is artificial and cannot model lateral

TABLE 1. Results of Earthquake Location by Three Different Procedures Described in the Text

	Origin, s	Latitude, N	Longitude, W	Depth, km	rms Error, s
Event 1					
Ray tracing	56.12	36°34.08'	121°09.36'	2.9	0.11
HYPO74	55.87	36°34.02'	121°09.98'	1.6	0.03
HYPO71	55.92	36°33.30'	121°10.93'	3.6	0.20
Event 2					
Ray tracing	16.97	36°35.16'	121°11.52'	6.8	0.11
HYPO74	16.65	36°35.51'	121°10.64'	6.1	0.03
HYPO71	16.85	36°34.66'	121°12.70'	6.2	0.14
Event 3					
Ray tracing	05.46	36°33.90'	121°08.34'	8.2	0.15
HYPO74	05.04	36°34.30'	121°07.06'	7.6	0.05
HYPO71	05.23	36°33.40'	121°09.61'	9.4	0.14

Event 1 took place on July 6, 1974, at 0403 (magnitude 3.1), event 2 on July 14, 1974, at 0030 (magnitude 2.1), and event 3 on July 21, 1974, at 0737 (magnitude 2.0).

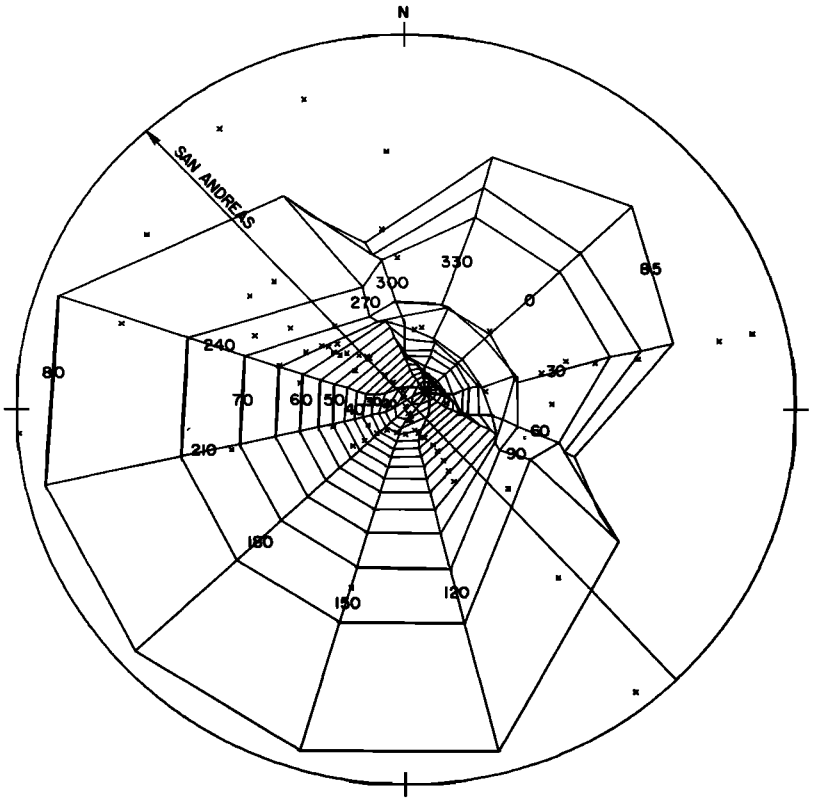

Fig. 4. Equidistant azimuthal surface projection of rays traced at 5° increments of takeoff angle and 30° increments of takeoff azimuth from event 2 (6.8-km-deep earthquake). Lines connect emergent points of rays of equal takeoff angle or azimuth. Crosses denote locations of stations for which P arrivals are reported. The circle is 40 km in radius and centered on the epicenter.

variations in the crust realistically. In the present study we will locate these earthquakes using seismic ray tracing in the manner described in the preceding sections. Since the surface geology is so complicated in this area, we did not apply station or elevation corrections, so that quoted depths are with respect to an elevation of 341 m at the reference point. Convergence was assumed when hypocenters found on successive iterations fell within 0.1 km of each other.

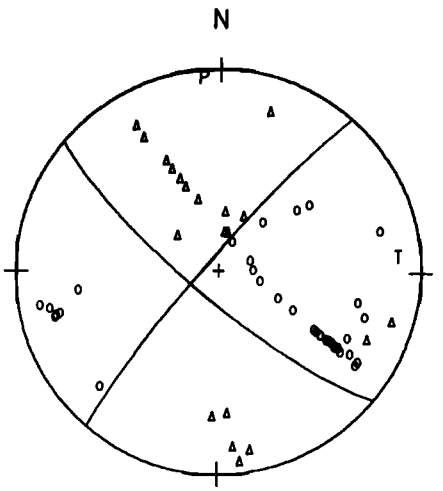

Fig. 5. Equal area azimuthal projection of lower hemisphere of focal sphere for event 2 (6.8-km-deep earthquake). P and T are pressure and tension axes, respectively. Rays are plotted at their antipodal points.

The three procedures for locating earthquakes described above may be abbreviated for reference purposes as 'HYP071', 'HYP074', and 'ray tracing'. We have applied these three procedures to the P arrivals observed for the three selected earthquakes in Bear Valley. The results are tabulated in Table 1, and the epicenter locations are shown in Figure 3. The 'HYP071' epicenter locations are 1 to 3 km to the west of the San Andreas fault. Both 'HYP074' and 'ray tracing' procedures located the epicenters near the San Andreas. In fact, the 'ray tracing' epicenters were within about 1 km of the surface trace of the San Andreas fault.

Since the reading error for P arrivals in the data used is about ± 0.03 s, the rms errors of about 0.1 s for the 'ray tracing' procedure reflect additional uncertainties about the structure and possibly very localized geological conditions beneath individual stations. Nevertheless, the model predicts travel times to stations at the same distance on either side of the fault that may differ by over a second. In a conventional hypocenter location of these events, this would lead to a systematic bias toward the faster west side of the fault. This is exactly what has been observed in the 'HYP071' procedure as shown in Figure 3. The small rms errors of the 'HYP074' procedure were achieved by specially tailored station corrections. Since no station

TABLE 2. Parameters of Fault Plane Solutions by Three Different Procedures Described in the Text

Procedure	Nodal Plane A		Nodal Plane B		P Axis		T Axis	
	Azimuth	Dip	Azimuth	Dip	Azimuth	Plunge	Azimuth	Plunge
Event 1								
Ray tracing	N63°W	79°SW	N29°E	84°NW	343°	86°	73°	78°
HYP074	N62°W	80°SW	N31°E	80°NW	345°	86°	74°	72°
HYP071								
Event 2								
Ray tracing	N51°W	79°SW	N41°E	84°NW	355°	86°	85°	78°
HYP074	N54°W	70°W	N36°E	90°	350°	75°	82°	76°
HYP071								
Event 3								
Ray tracing	N75°W	64°N	N10°E	80°W	240°	79°	144°	64°
HYP074	N87°W	90°	N 3°E	90°	228°	90°	138°	90°
HYP071	N70°W	86°S	N20°E	72°W	64°	74°	156°	80°

No entry indicates no data possible

corrections were applied to the 'ray tracing' procedure, it is difficult to compare the rms errors of it to those of the 'HYP074' procedure.

For the heterogeneous crustal model that we used for ray tracing, the effects on ray paths are dramatic, as shown in Figures 2 and 4. In Figure 2 are ray paths taken at 5° intervals of takeoff angle along an azimuth normal to the fault. This is the only azimuth along which the rays stay in a plane, so that Figure 2 is a true projection. The effect of structure on the ray paths is quite pronounced. Near-vertical rays to the west are refracted back across the fault to the east. Large shadow zones are produced. Not unexpectedly, the predicted ray path distortion is highly dependent on the location of the earthquake.

Figure 4 shows how the rays are distorted for the 6.8-km-deep event in full plan view. In this projection the lines connect emergent points of traced rays taken at 30° increments of takeoff azimuth and 5° intervals of takeoff angle. To the west of the fault the more homogeneous structure produces a symmetric distribution of rays. To the east there is considerable distortion.

It should be evident from these observations that this type of structure may produce pronounced differences in the location of rays projected on a focal sphere from positions calculated using a symmetric earth model and, hence, significant changes in focal mechanism determinations. In Figure 5 we show a focal mechanism solution for the 6.8-km-depth event based only on traced rays. Except for two inconsistencies the solution appears well determined. Similar determinations for the 2.9-km and 8.2-km-depth events, listed in Table 2, resulted in solutions with three and no inconsistencies, respectively. In view of uncertainties in some station polarities and in the assumed structure we find these results remarkably good. Fault plane solutions using the 'HYP071' and 'HYP074' procedures were also attempted, and results were also listed in Table 2. It is interesting to note that 'HYP074' procedure yields fault plane solutions similar to those of the 'ray tracing' procedure, whereas we failed to arrive at a solution for two of the three earthquakes using the 'HYP071' procedure.

In a qualitative way, Figures 2 and 4 also predict large variations in observed amplitudes. Since lines of equal takeoff angle and equal takeoff azimuth are plotted here, where there is a convergence of lines we would expect high amplitudes and where there are shadows low amplitudes. An examination of observed amplitudes and coda shows that there are large variations between stations. Detailed studies, however, cannot be carried out because amplitudes at most stations of the three selected events were clipped. Hence we need to examine smaller magnitude events. Furthermore, because of three different instrumental systems used in the USGS Bear Valley experiment, a considerable amount of work is necessary to obtain reliable station responses.

Discussion

For an assumed two-dimensional velocity model of the Bear Valley area, we find that hypocenters determined by seismic ray tracing are located near the San Andreas fault. Our confidence in these solutions is enhanced by the internal con-

sistency of P arrival time data in the rms sense and focal mechanism solutions.

It must be emphasized that the predicted ray-theoretical effects previously discussed are not slowly varying but are highly dependent on the spatial coordinates of the source. This suggests that small changes in location may produce large differences in travel times, which complicates the determination of station corrections and the evidence for P wave delays that may arise from dilatant volumes in areas of complex structure.

One limitation of the method is that because of difficulties in tracing rays with near-horizontal paths, we can only use upward traveling rays and hence require a dense network of close-in seismographs. In addition, if the models become too complicated, we are always faced with a uniqueness problem; i.e., two rays arriving at a station at the same time or quasi-linearity leading to more than one minimum in the source parameters. Fortunately, models tested to date appear to converge rapidly and uniquely.

In future work we hope to extend this technique by jointly determining structure and source parameters using inverse theory and seismic ray tracing. The inverse technique has already been successfully applied for the case of a homogeneous initial model by Aki and Lee [1976].

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