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# EFFECTS OF SELECTIVE FUSION ON THE THERMAL HISTORY OF THE EARTH'S MANTLE *

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A comparative study on the thermal history of the earth's mantle was made by numerical solutions of the heat equation including and excluding selective fusion of silicates. Selective fusion was approximated by melting in a multicomponent system and redistribution of radioactive elements. Effects of selective fusion on the thermal models are (1) lowering (by several hundred degrees centigrade) and stabilizing the internal temperature distribution, and (2) increasing the surface heat-flow. It was found that models with selective fusion gave results more compatible with observations of both present temperature and surface heat-flow. The results therefore suggest continuous differentiation of the earth's mantle throughout geologic time, and support the hypothesis that the earth's atmosphere, oceans, and crust have been accumulated throughout the earth's history by degassing and selective fusion of the mantle.

## 1. INTRODUCTION

Knowledge of the temperature within the earth is essential in understanding its dynamic behavior and the development of its surface features. This letter discusses a finding of Lee [1] that selective fusion of silicates can play a dominant role in the thermal history of the earth's mantle, and presents new results based on the recent determination of radioactivity in carbonaceous chondrites by Morgan and Lovering [2]. Its purpose is to show the difference in the development of internal temperature distribution and surface heat-flow between thermal models of the earth's mantle including and excluding selective fusion of silicates.

Thermal history of the earth is usually formulated as an initial-boundary-value problem which assumes *a priori* that the thermal state of the earth is given at some time past in its history, and the earth's surface temperature is known during all its subsequent history. To make the problem mathematically manageable, idealized models for the earth are used. But the validity of any given model can be easily tested. It must give a present temperature distribution within the limits deduced from geophysical evidence, and a surface heat-flow within the observed limits. Thermal models which disagree with the observations must be rejected. However, that a thermal model which agrees with the observations can be found does not necessarily imply that the actual thermal history of the earth must have developed exactly as the model. The thermal-history problem does not have a unique solution because a number of different assumptions may lead to the same results.

The scope of the present letter does not permit detailed discussions on the assumptions that enter into the thermal-history calculations and their effects on the results. Detailed reviews on the earth's thermal history have been given recently by Lee [1] and Lubimova [3]. It must be emphasized that the present work is a comparative study of thermal models that differ only in one respect: stationary versus moving heat sources. Consequently, the choice of all other assumptions is only of secondary importance.

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## 2. INITIAL CONDITIONS

In the present work, the starting point of the earth's thermal history is taken at a time after the earth was accreted and the core of the present size was formed, and the initial temperature is thus likely to be the melting point of iron [1]. This point of view is adopted from Ringwood's hypothesis on the origin of the earth [4], which assumes that the earth was formed from initially cold and unsorted conglomerations of cosmic dust. During the relatively short period of accretion (of the order of $10^8$ years), the growing earth was heated up as materials fell in, reaching about the melting temperature of iron. Liquid iron is heavier than the silicates, so it sank and formed the core. There is plenty of energy available in these processes [1, 3]: gravitational energy due to accretion – $25 \times 10^{38}$ ergs, and gravitational energy due to core formation – $2 \times 10^{38}$ ergs. Heat produced by short-lived radioactive elements can be ignored because the time interval between the solar-system nucleosynthesis and the earth's accretion is large compared to their half-lives ($10^9$ versus $10^6$ years).

Even though the earth's heat capacity is enormous ($7 \times 10^{34}$ ergs/$^{\circ}$K), gravitational energy due to accretion could have heated the whole earth from $0^{\circ}$K to $4 \times 10^4$ $^{\circ}$K. However, much of this energy must have been lost by radiation which is proportional to the 4th power of the surface temperature. For example, if the earth's surface is at $10^3$ $^{\circ}$K, it will radiate at a rate of $10^{34}$ ergs/year, and thus all the accretion energy could have been radiated away in 250000 years. The earth could not have passed through extremely high temperatures for it still retains some volatiles. This sets an upper limit on the earth's initial temperature not much above the melting point of iron [1].

After the earth was accreted and the core was formed, the main source of energy is the heat produced by radioactive decays of U, Th, and K – 1 to $2 \times 10^{38}$ ergs in 4.5 billion years depending on the radioactivity assumed [1]. Although energy dissipation due to tidal friction can be $0.4 \times 10^{38}$ ergs [3], this estimate depends on the history of the earth-moon system which is not well known. Furthermore, most of the tidal dissipation may have taken place in the oceans so that this source of energy is best ignored at present. Thermal evolution of the earth's mantle then depends not only on the amount of radioactive elements but also on their distribution during geologic history. Three possibilities exist: (1) radioactive elements remain uniformly distributed and undifferentiated, (2) selective fusion of silicates becomes active so that an initially uniform distribution of radioactive elements changes progressively into a concentrated distribution towards the earth's surface, and (3) differentiation had already occurred during the accretion and core formation so that radioactive elements were already concentrated in the upper mantle and subsequent differentiation is either insignificant or leads to even more concentrated radioactivity toward the surface.

Selective fusion of silicates has long been recognized as an important process within the earth. For example, Rubey's paper on the geologic history of sea water [5] strongly advocated the hypothesis that the earth's atmosphere, oceans, and crust have been accumulated slowly throughout geologic time by degassing and selective fusion of the mantle. Because the earth is a multicomponent system, melting occurs over a wide range of temperatures. The melting point of the lowest melting fraction may be several hundred degrees centrigrade lower than the highest melting component. Radioactive elements, water, and gases tend to be concentrated in early melts. Molten rock is lighter than the unmelted portion, so magma tends to rise and carry with it radioactive elements, water, and gases. Such differentiation by selective fusion and outgassing results in the progressive migration of radioactive elements, water, and gases toward the earth's surface in the course of geologic history.

## 3. THERMAL MODELS WITH SELECTIVE FUSION

In all thermal-history calculations prior to Lee [1], selective fusion of silicates is ignored and heat sources are assumed to be stationary (e.g. [6]). Consequently, Lee [1] has developed a thermal-history calculation for the earth which incorporates selective fusion of silicates and migration of radioactive elements. To make the problem mathematically manageable, several assumptions have been made. All thermal models studied are spherically symmetric. Selective fusion of silicates is approximated by melting in a multicomponent system and redistribution of radioactive elements by upward migration of magmas in a viscous medium. Latent heats for melting and freezing of a multicomponent system are allowed for in the computation of temperature across the transition boundaries. Moving heat sources and heat transfer due to penetrative convection are taken into account in the finite-difference equation, which is equivalent to the following heat equation:

$$\rho \frac{\partial(cT')}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(Kr^2 \frac{\partial T'}{\partial r}\right) + A - f\rho v \frac{\partial(cT')}{\partial r} , \quad (1)$$

where $\rho$ = density, $c$ = specific heat, $T'$ = adjusted temperature for latent heats if needed, $t$ = time, $r$ = radial distance, $K$ = thermal conductivity including radiative heat transfer, $A$ = heat production per unit volume per unit time, $f$ = ratio of mass of magma to the total mass, $v$ = velocity of magma rising in viscous medium.

Radioactive elements are redistributed by magmas rising like viscous globules in a more viscous medium with a velocity given by a Stokes' type formula ([1], p. 44, 120, 140):

$$v = \frac{a^2 g \Delta\rho}{3\eta} \propto a^2/\eta , \quad (2)$$

where $a$ = radius of magma globule, $g$ = gravitational acceleration, $\Delta\rho$ = density difference between magma and its surrounding medium, $\eta$ = viscosity of the surrounding medium.

Although $g$ and $\Delta\rho$ are reasonably well known, we are ignorant about the size of magma globules and the viscosity of the earth. The beauty of such a simple model, however, is that $a$ and $\eta$ work in opposite directions. If the assumed viscosity is high, $v$ is small, and radioactive elements migrate upward slowly. This tends to increase the local temperature and leads to larger rock melts. As a consequence, $\eta$ will decrease and $a$ will increase, thus increasing $v$. Hence eq. (2) is self-adjusting. In numerical computations the viscosity is assumed to be a function of temperature and melting temperature, and $a$ is assumed to be proportional to the amount of rock melt available. The proportionality constant is an input variable. Numerical experiments show that upward migration of radioactive elements plays a significant role in the thermal-history calculations only if $v$ is of the order of 1 mm/year or greater. A wide range of $a$ and $\eta$ will provide this velocity [1].

Melting temperatures for a few minerals and rocks have been measured up to a pressure of about 50 kilobars. This pressure is equivalent to a depth of 150 kilometers in the upper mantle, so that extensive extrapolation is necessary for melting temperatures at greater depths. For simplicity, the earth's models are assumed to consist of an 'iron' core and a 'silicate' mantle of two components: the low-melting 'basalt' and the high-melting 'dunite'. Their melting curves have been selected by Lee [1] on the basis of several estimates [7–9], as shown in figs. 1 and 2. We have no quantitative data on the partition of radioactive elements in selective fusion of silicates. Because radioactive elements occur mostly in grain boundaries, they tend to concentrate in early melts during the process of selective fusion. Hence, we assumed arbitrarily that

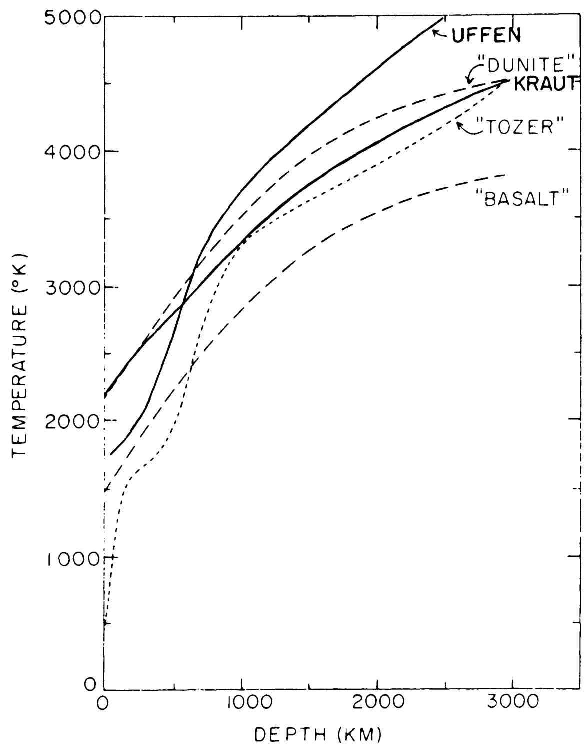


Fig. 1. Temperature estimates in the earth's mantle. Fusion curves based on Uffen's method [7] and on the Kraut-Kennedy equation [8] for forsterite are compared with the temperature distribution deduced from electrical conductivity data by Tozer [9]. Fusion curves of 'dunite' and 'basalt' are those selected by Lee [1].

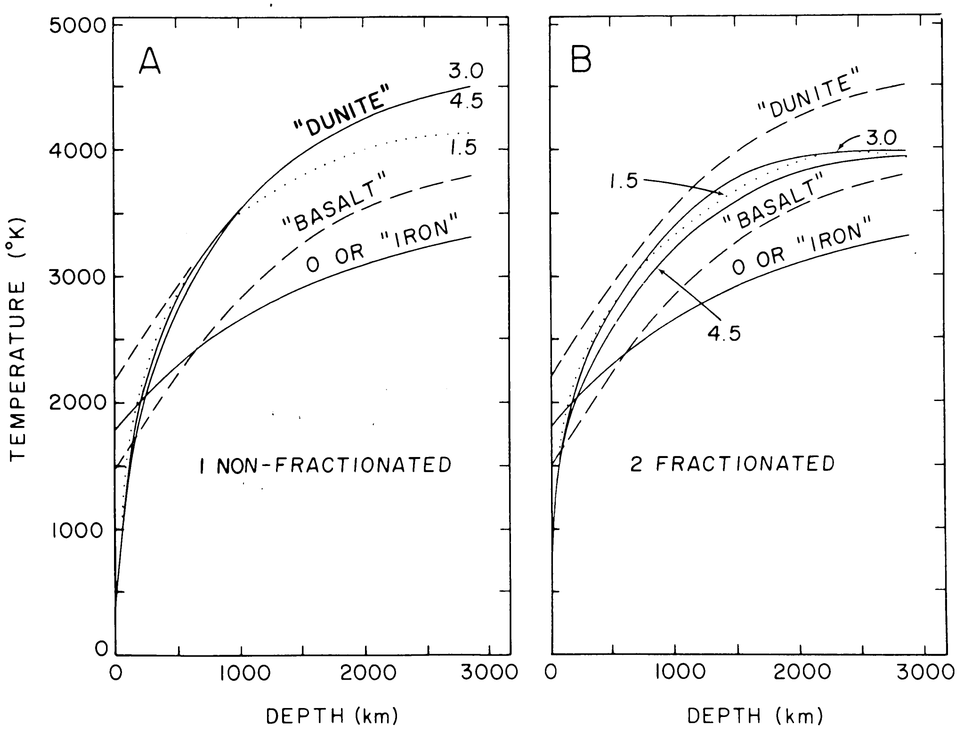


Fig. 2. Temperature developments in models 1 and 2 at 0, 1.5, 3.0 and 4.5 billion years after the earth was formed. Assumed melting curves for 'dunite' and 'basalt' are dashed. Data used for computation are identical for these two models, except that fractionation of radioactive elements is allowed in model 2 but not in model 1.

95% of the available radioactive elements are in the low-melting component 'basalt'.

Additional data needed in the thermal-history calculations for earth models with selective fusion are: $R$, $D$, $t_0$, $T(R,t)$, $\rho$, $c$, $T(r,t_0)$, $K$, $A$. The mean radius of the earth ($R$) is 6371 km; the depth to the core-mantle boundary ($D$) is at 2898 km; the age of the earth ($t_0$) is 4.5 billion years. From astrophysical and geological considerations ([1], pp. 14-16), the surface temperature of the earth ($T(R,t)$) may be taken as a constant: 0°C. Because the heat equation as formulated above does not permit density changes with time, we take the present density distribution ($\rho(r)$) as determined recently by Birch [10].

This assumption of density distribution is reasonable because the coefficient of thermal expansion of the mantle rocks is small, and density changes due to selective fusion are negligible. The specific heat ($c$) within the earth approaches the classical value at high temperature, and may therefore be taken as a constant: 1.3 J/g°C. The thermal conductivity is taken as a combination of lattice conduction and radiative heat transfer [11], and values of the parameters are based on recent works as reviewed in [1]. Radiative transfer is insignificant at low temperatures, but becomes of the same magnitude as the lattice conduction at 1500°K, and about six times greater at 3000°K.

The heat production term $A$ is considered to be entirely due to radioactive elements: U, Th, and K. Gravitational energy released on account of selective fusion is small compared with the radiogenic heat. For example, gravitational energy per unit area in fractionating a homogeneous mantle (95% 'dunite', and 5% 'basalt') of thickness ($H + h$) into all 'basalt' on the top is given by:

$$\Delta E = \tfrac{1}{2}\, g H h \Delta\rho\,, \tag{3}$$

where $g$ = gravitational acceleration, $H$ = thickness of 'dunite', $h$ = thickness of 'basalt', $\Delta\rho$ = density difference between 'dunite' and 'basalt'.

Assuming a mantle thickness of 3000 km and putting $g = 10^3$ cm/sec$^2$, and $\Delta\rho = 1$ g/cm$^3$, we have $\Delta E \simeq 2 \times 10^{18}$ ergs/cm$^2$. Taking the surface area of the earth to be $5 \times 10^{18}$ cm$^2$, the total gravitational energy released amounts to about $10^{37}$ ergs, or 5 to 10% of the radiogenic heat.

Several radioactivity models have been proposed: the ordinary chondrite, the Wasserburg, and the type I carbonaceous chondrite. The mean heat production for these models differs by less than a factor of two, so the choice of a particular radioactivity model is important but not critical [1]. Following arguments by Ringwood [4], we have assumed that the earth models have radioactive elements similar to type I carbonaceous chondrites, as recently determined by Morgan and Lovering [2] on a water- and carbon-free basis. Because radioactive elements do not seem to enter into the metal phase of the core, we assumed that all radioactive elements were in the mantle with a concentration of U = $2.22 \times 10^{-8}$, Th = $6.93 \times 10^{-8}$, and K = $1.08 \times 10^{-3}$ g/g.

The actual computations for solving these equations are involved and have been given in Lee [1]. The heat equation (1) is solved by the Crank-Nicolson implicit method, which guarantees numerical stability. MacDonald [6] used a much simpler finite-difference method which gave stable solutions only if:

$$\Delta t \leqslant \frac{(\Delta r)^2}{2\kappa}, \qquad (4)$$

where $\Delta t$ is the time increment, and $\Delta r$ is the radial increment in approximating the heat equation. The thermal diffusivity is $\kappa$, which is defined as $K/\rho c$. MacDonald's technique therefore requires very small time increments, and this leads to a large amount of computing time as well as to an accumulation of round-off errors. The method used in this paper eliminates most of these problems by having a more complex computational scheme.

## 4. DISCUSSION OF RESULTS

The present temperature distribution in the earth's mantle is not well known. However, several independent arguments have led to similar estimates and the best estimate is perhaps that by Tozer as shown in fig. 1. The present mean observed surface heat-flow is fairly well determined to be 1.5 ± 10% $\mu$cal/cm$^2$sec [12]. The present temperature estimate and the observed surface heat-flow do not enter into the assumptions of the thermal-history calculations. Any calculation for thermal history will give a computed present temperature and surface heat-flow. We can then compare them with the observed values and eliminate those models that do not agree.

The first two models considered were identical except for the fractionation of radioactive elements. No redistribution of radioactive elements was allowed in model 1, and it is therefore similar to the previously published thermal models. However, radioactive elements were allowed to migrate upward in model 2 whenever selective fusion occurred. Fig. 2 shows the

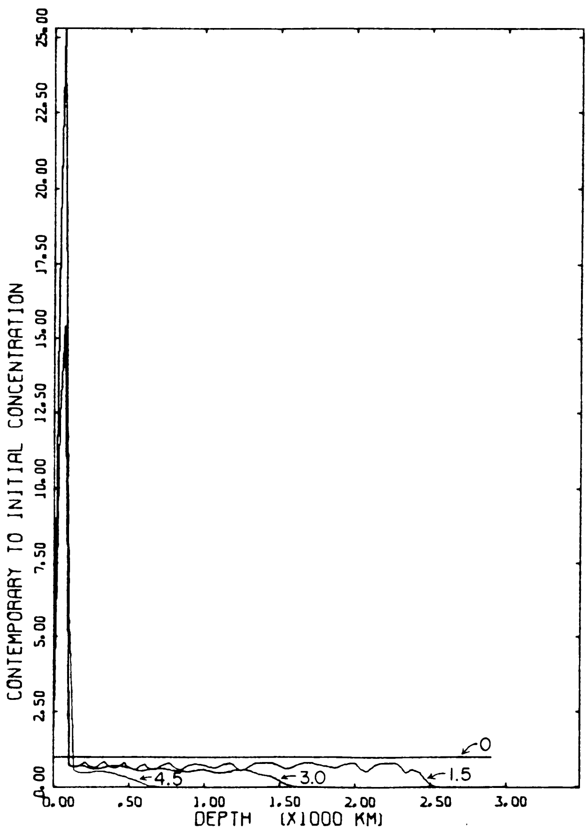


Fig. 3. Ratio of contemporary to initial concentration of radioactive elements as a function of depth at 0, 1.5, 3.0, and 4.5 billion years after the earth was formed.

contrast of temperature developments between model 1 (non-fractionated) and model 2 (fractionated). The initial temperature was identical to the fusion curve of 'iron', and the radioactivity in the mantle was uniform at time 0. Because fractionation of radioactive elements was not allowed in model 1, temperature rose rapidly and approached the melting curve of 'dunite' at 1.5 billion years. After 4.5 billion years, the temperature in most of the mantle was at the melting curve of 'dunite'. This model must be rejected because (1) it implies that the mantle is largely molten, and (2) it yields a surface heat-flow about half of that observed. When fractionation of radioactive elements was allowed as in model 2, the rapid rise of temperature was halted because upward migration of radioactive elements has depleted the heat sources in the lower mantle and increased the surface heat loss. The temperature in this fractionated model was below the melting curve of 'dunite' at all times, and was stabilized after 1.5 billion years or since 3 billion years ago.

Upward migration of radioactive elements in model 2 is illustrated in fig. 3. If no fractionation occurs, the ratio of concentration of radioactive elements at any time to the initial concentration will be 1, as shown here at the initial time 0. Fractionation, however, started shortly and depleted the deep mantle of radioactive elements at 1.5 billion years. This process continued so that all radioactive elements were in the outer 600 kilometers of the earth at 4.5 billion years, i.e., at present. Some upward concentration of radioactive elements in the outer 100 kilometers took place early in the earth's history. This happened because the melting point of 'iron' at low pressure is higher than that of 'basalt'.

Fig. 4 shows the present temperature distribution from three thermal models. Model 3 is identical to models 1 and 2 except that all radioactive elements were concentrated in the outer 1000 kilometers at the initial time 0, and no fractionation of radioactive elements was allowed. As noted previously model 1 (non-fractionated) gives a present temperature distribution identical to the fusion curve of 'dunite' below 1000 kilometers, and model 2 (fractionated) gives a present temperature close to that estimated by Tozer. Model

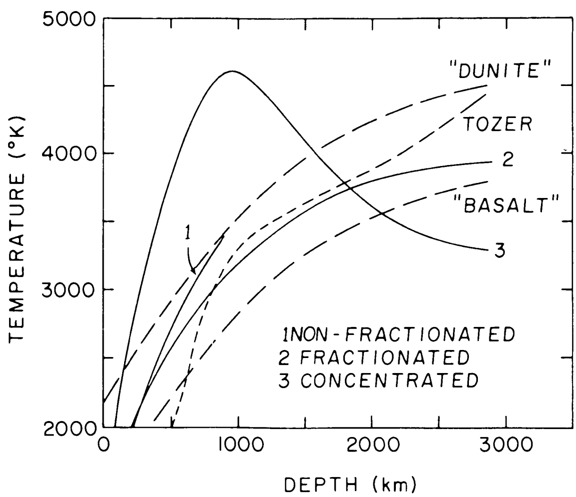


Fig. 4. Present temperature distribution for models 1, 2, and 3. Assumed melting curves for 'dunite' and 'basalt' are dashed. Temperature distribution deduced from electrical conductivity data is marked 'Tozer'.

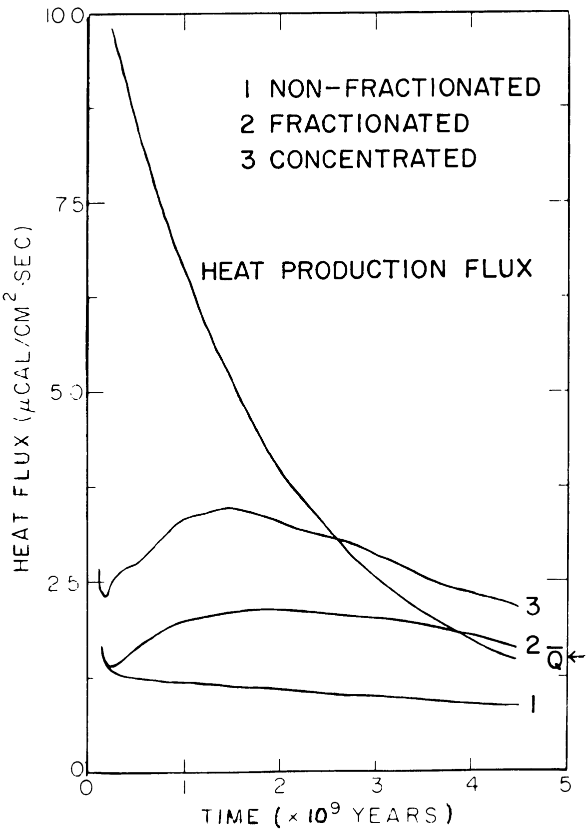


Fig. 5. Surface heat-flow for models 1, 2, and 3. The heat-production flux is computed from the rate of radiogenic heat production divided by the earth's surface area. The present observed surface heat-flow is marked $\overline{Q}$.

3, however, gives a present temperature very different from that of 'Tozer' and implies that the outer 1500 kilometers or so of the mantle is molten. Nevertheless this difficulty can be avoided if more efficient heat loss than thermal conduction (e.g., extensive volcanism) exists. These results show that upward migration of radioactive elements can play an important role in the earth's thermal history. To obtain a reasonable present temperature, models with moving heat sources are superior to models with stationary heat sources, either uniformly distributed or concentrated in the upper mantle.

Another test for thermal models is to compare the computed surface heat-flow with that observed. Fig. 5 shows the computed surface heat-flow versus time. The heat-production flux of the earth models is included for comparison; and it is computed from the rate of radiogenic heat production divided by the earth's surface area. This flux decreases with time because of the exponential decay of radioactive elements. The present observed surface heat-flow is 1.5 $\mu$cal/cm$^2$sec and is marked by $\bar{Q}$. Only model 2 yields acceptable surface heat-flow at present. Model 1 gives only half of the observed heat-flow, whereas model 3 exceeds the observed heat-flow by about 50%. Model 2 also gives a surface heat-flow that is fairly uniform throughout the earth's history. If radioactive elements were concentrated in the upper 1000 kilometers of the mantle at the initial time, as in model 3, the surface heat-flow during the first two billion years would have been twice as large as that at present. Because thermal conductivity of rocks is fairly constant for temperatures up to 1000$^\circ$K, increasing surface heat-flow can be accomplished only by increasing the temperature gradient with depth in the crust and upper mantle. For this reason model 3 implies far more extensive volcanism and related geothermal activity in the past than at present.

Convective heat transfer in the fractionated models is not significant because the amount of rock melt is small in mass and slow in motion. The question of large-scale convection in the earth's mantle, however, still remains unsolved. A model with large-scale convection has been formulated and programmed in Lee [1] by numerically solving the heat equation, coupled with the equation of continuity and the equations of motion. Unfortunately, the large amount of computing time required does not make such calculations feasible at present. We need a computer hundreds of times faster than the best we have today. Alternatively, a better numerical scheme for solving the field equations must be developed.

## REFERENCES

[1] W. H. K. Lee, Thermal history of the earth, Ph.D. dissertation, University of California, Los Angeles (1967) 344 pp.
[2] J. W. Morgan and J. F. Lovering, Uranium and thorium abundances in carbonaceous chondrites, Nature 213 (1967) 873.
[3] E. A. Lubimova, Theory of thermal state of the earth's mantle, in: The Earth's Mantle, ed. T. F. Gaskell (Academic Press, London, 1967) p. 231.
[4] A. E. Ringwood, The chemical composition and origin of the earth, in: Advances in Earth Science, ed. P. M. Hurley (Massachusetts Institute of Technology Press, Cambridge, 1966) p. 287.
[5] W. W. Rubey, Geologic history of sea water, Geol. Soc. Am. Bull. 62 (1951) 1111.
[6] G. J. F. MacDonald, Calculations on the thermal history of the earth, J. Geophys. Res. 64 (1959) 1967.
[7] R. J. Uffen, A method of estimating the melting-point gradient in the earth's mantle, Trans. Am. Geophys. Union 33 (1952) 893.
[8] E. A. Kraut and G. C. Kennedy, New melting law at high pressure, Phys. Rev. 151 (1966) 668.
[9] D. C. Tozer, The electrical properties of the earth's interior, Phys. Chem. Earth 3 (1959) 414.
[10] F. Birch, Density and composition of mantle and core, J. Geophys. Res. 69 (1964) 4377.
[11] S. P. Clark, Radiative transfer in the earth's mantle, Trans. Am. Geophys. Union 38 (1957) 931.
[12] W. H. K. Lee and S. Uyeda, Review of heat flow data, in: Terrestrial Heat Flow, ed. W. H. K. Lee, Am. Geophys. Union, Geophysical Monograph 8 (1965) 87.