# Letters

## A Spherical Harmonic Analysis of the Earth's Topography[1]

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In 1964–1965 estimates were made of the percentage of land, average land elevation, and average ocean depth in each $1° \times 1°$ square and of percentages of lake and ice and average lake depth and ice thickness in each $5° \times 5°$ square. These estimates were made visually, and the following maps were used:

1. Chart of the World, No. 15254, 2nd edition, 1961, U. S. Naval Oceanographic Office, scale 1:12,000,000 at the equator, Mercator projection; main contour interval, 500 fathoms.
2. The World, Series 1101, 9th edition, 1964, U. S. Army Map Service, scale 1:11,000,000 approximately, Mercator projection; main contour interval, 1000 meters.
3. Arktika (Arctic), 1960, Main Administration in Geodesy and Cartography of the U.S.S.R. Geological Committee, scale 1:8,000,000, polar projection; main contour interval, 1000 meters.
4. Antarctica, 1962, American Geographical Society, New York, scale 1:3,000,000, polar projection; main contour interval, 1000 meters.

In some areas of rough topography these maps were supplemented by larger-scale maps.

The estimates of percentages and elevations for $1° \times 1°$ squares were made independently by two assistants and were rechecked when percentage estimates disagreed by more than 10% or elevation estimates disagreed by more than 250 meters. Transcription and card punching of estimates were also duplicated, and computer check programs were used to minimize errors. These values for $1° \times 1°$ squares were averaged together to form the mean values for $5° \times 5°$ squares which were used for the later stages of the analysis.

Sources of error in the mean elevations are

(1) Survey error: errors in the original soundings over the oceans, or in the triangulation plus photogrammetry or barometry on the land.

(2) Error of representation: the error arising from insufficient survey data.

(3) Map compilation error: discrepancies between maps and survey data.

(4) Contour interpolation error: the error arising from deviation of the actual topographic slope from a straight line between adjacent contour lines.

(5) Estimation error: errors arising from the visual estimation procedure.

Two tests of the estimates against external standards were made. The first was to re-estimate mean elevations for 100 randomly selected $1° \times 1°$ squares on the largest-scale maps available. Since both large- and small-scale maps were ultimately based on the same survey data, this comparison was a test of errors (3), (4), and (5). The mean absolute difference between estimates was 232 meters for the entire 100 and 135 meters for the 33 best-surveyed. The second test was to re-estimate from a more recent map, the U.S.S.R. Geological Committee 1964 Pacific, scale 1:10,000,000; main contour interval 500 meters. Mean elevation for ten $5° \times 5°$ squares was estimated from this map in areas more than 5° distant from any bathymetry known to be used in the U. S. Naval Oceanographic Office chart 15254, 1961. Hence this comparison was a test mainly of error (2). The mean absolute difference between estimates was 207 meters, and the rms ± 250 meters.

Of the five errors, (1), (3), and (5) should be negligible—(3) because at the scales used a $5° \times 5°$ square is of the order of a 4-cm

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TABLE 1. Spherical Harmonic Coefficients of Topography (in meters)
$n$ = order of spherical harmonic; $m$ = degree of spherical harmonic; $s$ = total topography; $o$ = oceanic topography; $c$ = continental topography; $r$ = equivalent rock topography; $b$ = Bruins' total topography.

| | | $s$ | | $o$ | | $c$ | | $r$ | | $b$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $m$ | $A_n^m$ | $B_n^m$ | $A_n^m$ | $B_n^m$ | $A_n^m$ | $B_n^m$ | $A_n^m$ | $B_n^m$ | $A_n^m$ | $B_n^m$ |
| 0 | 0 | −2300 | | −2520 | | 220 | | −1340 | | −2367 | |
| 1 | 0 | 328 | | 305 | | 24 | | 226 | | 668 | |
| 1 | 1 | 777 | 538 | 686 | 438 | 91 | 100 | 508 | 361 | 594 | 400 |
| 2 | 0 | 339 | | 272 | | 67 | | 214 | | 526 | |
| 2 | 1 | 401 | 343 | 402 | 275 | −1 | 68 | 243 | 246 | 344 | 344 |
| 2 | 2 | −564 | −140 | −487 | −186 | −76 | 46 | −371 | −64 | −432 | −95 |
| 3 | 0 | −110 | | −42 | | −68 | | −76 | | −144 | |
| 3 | 1 | −78 | 158 | −49 | 85 | −29 | 73 | −60 | 116 | −156 | 94 |
| 3 | 2 | −540 | 547 | −421 | 487 | −119 | 60 | −376 | 358 | −446 | 468 |
| 3 | 3 | 141 | 803 | 140 | 754 | 2 | 49 | 86 | 507 | 124 | 574 |
| 4 | 0 | 217 | | 188 | | 29 | | 125 | | 317 | |
| 4 | 1 | −192 | −191 | −170 | −119 | −22 | −72 | −126 | −131 | −220 | −232 |
| 4 | 2 | −469 | 128 | −339 | 120 | −129 | 8 | −334 | 84 | −398 | 71 |
| 4 | 3 | 462 | −105 | 420 | −32 | 43 | −73 | 298 | −92 | 381 | −139 |
| 4 | 4 | −128 | 670 | −142 | 575 | 14 | 95 | −71 | 446 | −56 | 482 |
| 5 | 0 | −253 | | −184 | | −69 | | −169 | | −539 | |
| 5 | 1 | −58 | −44 | −70 | −41 | 13 | −3 | −35 | −40 | −47 | −102 |
| 5 | 2 | −76 | −13 | −32 | −114 | −44 | −18 | −69 | −88 | 0 | −142 |
| 5 | 3 | 207 | −9 | 165 | 71 | 42 | −80 | 140 | −38 | 133 | 42 |
| 5 | 4 | 659 | −20 | 563 | 34 | 96 | −54 | 437 | −34 | 520 | −72 |
| 5 | 5 | −90 | 363 | −63 | 280 | −27 | 82 | −66 | 257 | −48 | 210 |
| 6 | 0 | 77 | | 34 | | 42 | | 52 | | 229 | |
| 6 | 1 | −20 | −150 | −4 | −65 | −16 | −86 | −19 | −109 | 19 | −141 |
| 6 | 2 | −34 | −123 | −50 | −110 | 16 | −13 | −11 | −77 | 38 | −146 |
| 6 | 3 | 107 | 217 | 93 | 244 | 15 | −27 | 74 | 123 | 60 | 181 |
| 6 | 4 | 335 | −223 | 244 | −184 | 91 | −39 | 241 | −151 | 201 | −179 |
| 6 | 5 | −136 | −260 | −95 | −281 | −41 | 21 | −101 | −149 | −123 | −198 |
| 6 | 6 | 69 | 41 | 108 | 32 | −39 | 10 | 27 | 30 | 32 | 27 |
| 7 | 0 | −88 | | −88 | | −0 | | −48 | | −284 | |
| 7 | 1 | 27 | 106 | −88 | 72 | 28 | 35 | 22 | 69 | 17 | 147 |
| 7 | 2 | 131 | 7 | 58 | 24 | 73 | −18 | 103 | −1 | 166 | 23 |
| 7 | 3 | −58 | 45 | −84 | 36 | 26 | 8 | −29 | 31 | −85 | 12 |
| 7 | 4 | −236 | 12 | −276 | 13 | 40 | −1 | −127 | 8 | −263 | 32 |
| 7 | 5 | 2 | −37 | 18 | −85 | −16 | 49 | −7 | −4 | 16 | −12 |
| 7 | 6 | −84 | −155 | −51 | −148 | −34 | −6 | −65 | −98 | −55 | −108 |
| 7 | 7 | −75 | −145 | −38 | −99 | −36 | −47 | −60 | −107 | −14 | −94 |
| 8 | 0 | −28 | | −49 | | 21 | | −13 | | 4 | |
| 8 | 1 | −26 | −26 | −28 | 2 | 2 | −29 | −15 | −15 | 0 | −25 |
| 8 | 2 | 127 | −2 | 95 | 21 | 32 | −23 | 97 | −6 | 131 | 36 |
| 8 | 3 | 23 | 48 | 14 | 26 | 10 | 21 | 24 | 41 | 38 | 54 |
| 8 | 4 | −60 | 72 | −60 | 78 | 0 | −6 | −34 | 40 | −21 | 82 |
| 8 | 5 | −96 | −20 | −69 | −81 | −27 | 61 | −67 | 10 | −72 | 3 |
| 8 | 6 | 116 | 61 | 128 | 46 | −12 | 15 | 68 | 43 | 90 | 72 |
| 8 | 7 | 259 | 80 | 198 | 103 | 60 | −23 | 185 | 41 | 165 | 38 |
| 8 | 8 | −153 | −212 | −118 | −184 | −35 | −27 | −109 | −140 | −112 | −133 |

square, much more than any reasonable plotting error or symbol displacement, and (5) because the average spacing of 500-fathom contour lines of about 2° in well-surveyed areas provides sufficient detail for the estimates. Some of (1), survey error, does exist though, according to *Emery and Shekhvatov* [1966], but they conclude that the bathymetry is 'as good as can be expected.'

To obtain numerical estimates of (2), error of representation, and (4), contour interpolation error, in a manner similar to that applied to gravimetry [*Kaula*, 1959], we require short-range autocovariance data. The sum of squares of column *s* in Table 5 through degree 18 gives $6.10 \times 10^6$ m² as the variance $\langle(h - \bar{h})^2\rangle$ for 10° × 10° mean elevations; through degree 35, we get $6.44 \times 10^6$ m² for 5° × 5° means. If we assume linear change with distance for short-range covariance $\mathrm{Cov}_s(h)$ of point elevations, using the method of *Kaula* [1959, pp. 63–64] we get for the autocovariance

$$\mathrm{Cov}_s\,(h) = 6.76 \times 10^6[1 - 0.01875s] \qquad (1)$$

for *s* in degrees. A crude check is obtainable from the spacing of contour lines, since, for a distance *d*,

$$E_d(\Delta h^2) = 2[\mathrm{Var}\,(h) - \mathrm{Cov}_d\,(h)] \qquad (2)$$

The rms interval between 500-fathom (915-m) contours is about 3°, which, using 915 m for $\Delta h$ and $6.76 \times 10^6$ for $\mathrm{Var}(h)$ in (2), gives $6.34 \times 10^6$ for $\mathrm{Cov}_3(h)$ compared with $6.38 \times 10^6$ m² from (1); this is close enough. For the average mean square interpolation error of the mean elevation of a segment of length *b* between contours an interval *d* apart, we get, after some mundane algebra,

$$E\{\epsilon^2(h_b)\} = V_0k(d - b)/3 \qquad (3)$$

Using 6.76 for $V_0$, 0.01875 for *k*, 3 for *d*, and 1 for *b*, we get ±290 meters for the rms interpolation error of a 1° segment, which agrees well enough with the 232 meters for the mean error of the sample of 100 1° × 1° squares.

The average contour spacing will also influence the effect of interpolation errors in the 1° × 1° means on the 5° × 5° means because it will set the limit on significant correlation of errors. If we take the 3° spacing as effectively breaking up the 5° × 5° square into four independent blocks, the effect of interpolation error on the 5° × 5° mean elevations will be about ±140 meters.

The mean square error of representation for an average spacing *s* between sounding tracks is tedious to calculate; let us assume that it will be about 0.7 times the error for point values of spacing *s* [*Kaula*, 1959, p. 135]. For the maximum spacing *s* of about 10°, the error of representation of 10° × 10° means is about $0.7[6.76 - 6.10] \times 10^6 \approx 0.46 \times 10^6$ m², or ±680 meters. For a spacing *s* of about 5° it will be ±470 meters. This error is comparable to the 250-meter rms discrepancy obtained in the comparison with the more recent and detailed U.S.S.R. maps, and so the assumed covariance appears to be on the pessimistic side.

For smaller spacings, the error of the 5° mean on the linear covariance assumption will be roughly ±470*s*/5 meters.

TABLE 2. Spherical Harmonic Coefficients of the Ocean Function

| | | Present Paper, 5° × 5° squares | | *Munk and MacDonald* [1960] | |
|---|---|---|---|---|---|
| $n$ | $m$ | $A_n^m$ | $B_n^m$ | $A_n^m$ | $B_n^m$ |
| 0 | 0 | 0.709 | | 0.714 | |
| 1 | 0 | −0.051 | | −0.123 | |
| 1 | 1 | −0.144 | −0.079 | −0.108 | −0.055 |
| 2 | 0 | −0.040 | | −0.058 | |
| 2 | 1 | −0.053 | −0.068 | −0.039 | −0.061 |
| 2 | 2 | 0.051 | 0.002 | 0.077 | −0.005 |
| 3 | 0 | 0.036 | | 0.044 | |
| 3 | 1 | 0.035 | −0.046 | 0.046 | −0.039 |
| 3 | 2 | 0.074 | −0.109 | 0.125 | −0.179 |
| 3 | 3 | −0.011 | −0.122 | −0.017 | −0.252 |
| 4 | 0 | −0.016 | | −0.026 | |
| 4 | 1 | 0.035 | 0.016 | 0.041 | 0.025 |
| 4 | 2 | 0.097 | −0.040 | 0.175 | −0.043 |
| 4 | 3 | −0.060 | −0.001 | −0.144 | 0.007 |
| 4 | 4 | 0.033 | −0.153 | −0.069 | −0.406 |
| 5 | 0 | 0.056 | | 0.101 | |
| 5 | 1 | 0.001 | 0.008 | −0.008 | 0.018 |
| 5 | 2 | 0.060 | 0.020 | 0.097 | 0.052 |
| 5 | 3 | −0.039 | −0.012 | −0.107 | −0.036 |
| 5 | 4 | −0.118 | 0.027 | −0.363 | 0.106 |
| 5 | 5 | −0.002 | −0.074 | 0.000 | −0.257 |
| 6 | 0 | −0.007 | | −0.033 | |
| 6 | 1 | 0.007 | 0.017 | 0.009 | 0.020 |
| 6 | 2 | 0.027 | 0.001 | 0.033 | −0.006 |
| 6 | 3 | −0.003 | −0.032 | 0.002 | −0.075 |
| 6 | 4 | −0.050 | −0.036 | −0.110 | 0.091 |
| 6 | 5 | 0.028 | 0.027 | 0.110 | 0.115 |
| 6 | 6 | −0.009 | −0.024 | −0.012 | −0.078 |
| 7 | 0 | 0.025 | | 0.051 | |
| 7 | 1 | 0.002 | −0.025 | −0.006 | −0.035 |
| 7 | 2 | −0.017 | −0.006 | −0.049 | −0.002 |
| 7 | 3 | 0.010 | −0.018 | 0.043 | −0.032 |
| 7 | 4 | 0.026 | −0.003 | 0.109 | −0.026 |
| 7 | 5 | −0.005 | 0.032 | −0.021 | 0.119 |
| 7 | 6 | 0.007 | 0.039 | 0.031 | 0.163 |
| 7 | 7 | 0.013 | 0.049 | 0.043 | 0.224 |
| 8 | 0 | 0.012 | | 0.010 | |
| 8 | 1 | 0.001 | 0.009 | −0.002 | 0.027 |
| 8 | 2 | −0.012 | 0.004 | −0.020 | 0.024 |
| 8 | 3 | −0.011 | −0.015 | −0.046 | −0.027 |
| 8 | 4 | 0.002 | −0.014 | 0.000 | −0.052 |
| 8 | 5 | 0.024 | 0.006 | 0.116 | −0.002 |
| 8 | 6 | −0.014 | −0.013 | −0.073 | −0.115 |
| 8 | 7 | −0.047 | −0.023 | −0.243 | −0.122 |
| 8 | 8 | −0.025 | 0.037 | −0.136 | 0.175 |

TABLE 3. RMS Variability of Topography

$n$ = order of spherical harmonic; $a$ = total topography by *Prey* [1922]; $b$ = total topography by *Bruins*; $s$ = total topography; $o$ = oceanic topography; $c$ = continental topography; $r$ = equivalent rock topography; $S$, $O$, $C$, $R$ correspond to $s$, $o$, $c$, $r$ by multiplying $[n(n+1)]^{1/2}$.

| $n$ | $a$ | $b$ | $s$ | $o$ | $c$ | $r$ | $S$ | $O$ | $C$ | $R$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 2456 | 2367 | 2300 | 2520 | 220 | 1340 | 0 | 0 | 0 | 0 |
| 1 | 1104 | 979 | 1000 | 869 | 137 | 663 | 1414 | 1229 | 194 | 937 |
| 2 | 676 | 841 | 854 | 764 | 131 | 554 | 2093 | 1871 | 320 | 1358 |
| 3 | 867 | 905 | 1140 | 1007 | 176 | 746 | 3948 | 3488 | 609 | 2585 |
| 4 | 722 | 875 | 1023 | 858 | 199 | 685 | 4577 | 3836 | 891 | 3062 |
| 5 | 564 | 815 | 842 | 698 | 187 | 573 | 4614 | 3824 | 1024 | 3136 |
| 6 | 260 | 509 | 599 | 531 | 156 | 396 | 3880 | 3442 | 1009 | 2563 |
| 7 | 173 | 481 | 397 | 381 | 131 | 255 | 2970 | 2851 | 983 | 1907 |
| 8 | 96 | 338 | 445 | 388 | 120 | 306 | 3778 | 3294 | 1016 | 2593 |
| 9 | 108 | 384 | 427 | 373 | 126 | 288 | 4047 | 3537 | 1198 | 2734 |
| 10 | 99 | 373 | 391 | 350 | 104 | 258 | 4103 | 3676 | 1090 | 2706 |
| 11 | 56 | 289 | 297 | 264 | 91 | 201 | 3409 | 3029 | 1051 | 2304 |
| 12 | 42 | 247 | 313 | 272 | 91 | 210 | 3910 | 3402 | 1140 | 2622 |
| 13 | 55 | 271 | 299 | 257 | 82 | 199 | 4028 | 3465 | 1099 | 2690 |
| 14 | 52 | 293 | 329 | 307 | 71 | 210 | 4766 | 4447 | 1030 | 3043 |
| 15 | 48 | 243 | 249 | 316 | 70 | 166 | 3864 | 3353 | 1092 | 2575 |
| 16 | 33 | 239 | 202 | 178 | 72 | 136 | 3325 | 2930 | 1182 | 2239 |
| 17 | | 275 | 318 | 275 | 74 | 213 | 5556 | 4809 | 1290 | 3725 |
| 18 | | 212 | 204 | 191 | 67 | 134 | 3764 | 3524 | 1241 | 2478 |
| 19 | | 156 | 169 | 156 | 63 | 115 | 3293 | 3044 | 1227 | 2233 |
| 20 | | 186 | 200 | 182 | 49 | 131 | 4104 | 3733 | 997 | 2691 |
| 21 | | 192 | 164 | 139 | 62 | 115 | 3519 | 2998 | 1333 | 2466 |
| 22 | | 175 | 167 | 146 | 66 | 116 | 3747 | 3291 | 1488 | 2610 |
| 23 | | 143 | 166 | 136 | 61 | 118 | 3909 | 3202 | 1429 | 2766 |
| 24 | | 163 | 164 | 151 | 57 | 108 | 4015 | 3694 | 1384 | 2644 |
| 25 | | 156 | 177 | 165 | 49 | 116 | 4512 | 4218 | 1256 | 2969 |
| 26 | | 186 | 154 | 142 | 60 | 105 | 4076 | 3774 | 1580 | 2771 |
| 27 | | 135 | 143 | 125 | 53 | 101 | 3943 | 3434 | 1457 | 2764 |
| 28 | | 135 | 126 | 115 | 40 | 84 | 3596 | 3272 | 1129 | 2396 |
| 29 | | 127 | 137 | 135 | 42 | 88 | 4054 | 3973 | 1248 | 2604 |
| 30 | | 156 | 123 | 112 | 51 | 85 | 3761 | 3402 | 1561 | 2596 |
| 31 | | 101 | 120 | 105 | 51 | 85 | 3791 | 3317 | 1595 | 2679 |
| 32 | | | 120 | 109 | 47 | 84 | 3885 | 3540 | 1522 | 2724 |
| 33 | | | 116 | 104 | 39 | 79 | 3887 | 3485 | 1322 | 2652 |
| 34 | | | 124 | 112 | 50 | 85 | 4289 | 3871 | 1722 | 2927 |
| 35 | | | 114 | 96 | 44 | 81 | 4038 | 3421 | 1546 | 2871 |

The 5° × 5° means were harmonically analyzed to obtain the coefficients $A_n^m$ and $B_n^m$ of real surface spherical harmonics $Z_{nmc}$ and $Z_{nms}$ normalized so that

$$\int_{\text{sphere}} \begin{Bmatrix} Z_{nmc} \\ Z_{nms} \end{Bmatrix}^2 d\sigma = 4\pi \tag{4}$$

where

$$\left.\begin{matrix} Z_{nmc} \\ Z_{nms} \end{matrix}\right\} = \bar{P}_{nm}(\cos\theta)\left\{\begin{matrix} \cos m\lambda \\ \sin m\lambda \end{matrix}\right. \tag{5}$$

$\bar{P}_{nm}$ is the normalized Legendre associated function, $\theta$ is the colatitude, and $\lambda$ is the longitude.

The analysis was carried to $n = 36$. The resulting cosine $A_n^m$ and sine $B_n^m$ coefficients up to degree 8 are given in Tables 1 and 2. Also calculated as a measure of contribution of different wavelengths is the rms variability of the topography by wave number $n$,

$$\sigma_n = \left[\sum_{m=0}^{n} (A_n^{m2} + B_n^{m2})\right]^{1/2} \tag{6}$$

which is invariant under rotation. The resulting values are given in Table 3.

To estimate the errors in the normalized coefficients $A_n^m$, $B_n^m$, let us assume that they arise from (1) independent interpolation errors

TABLE 4. RMS Discrepancy in Spherical Harmonic Coefficients between Two Different Analyses

| | Total Topography (*Lee versus Bruins*) | | | Ocean Function (*Lee versus Munk and MacDonald*) | | |
|---|---|---|---|---|---|---|
| $n$ | $C_n$, m | $\Delta C_n$, m | $\Delta C_n/C_n$, % | $C_n$ | $\Delta C_n$ | $\Delta C_n/C_n$, % |
| 0 | 2300 | 67 | 2.9 | 0.709 | 0.005 | 0.7 |
| 1 | 576 | 237 | 41.1 | 0.099 | 0.048 | 48.5 |
| 2 | 381 | 107 | 28.1 | 0.048 | 0.016 | 33.3 |
| 3 | 430 | 106 | 24.7 | 0.072 | 0.059 | 81.9 |
| 4 | 342 | 88 | 25.7 | 0.067 | 0.099 | 148. |
| 5 | 254 | 121 | 47.6 | 0.051 | 0.100 | 196. |
| 6 | 166 | 64 | 38.6 | 0.025 | 0.055 | 220. |
| 7 | 103 | 60 | 58.3 | 0.023 | 0.067 | 291. |
| 8 | 108 | 38 | 35.2 | 0.020 | 0.079 | 395. |

of ±290 meters for $4\pi/(3\pi/180)^2 = 4580$ 3° × 3° *squares* (not 3° latitude × 3° longitude rectangles) and (2) independent errors of representation for squares of side length $s$ corresponding to the spacing $s$ of sounding tracks or other elevation control. Assuming 0.20 of the world to have spacing $s$ of 10°, 0.20 a spacing of 5°, 0.20 a spacing of 2°, and 0.40 a spacing of 1°, we get for the standard deviation $D$ of $A_n^m$ or $B_n^m$

$$D\begin{Bmatrix} A_n^m \\ B_n^m \end{Bmatrix} = \left[\frac{290^2}{4580} + 0.20\,\frac{680^2}{410} + \left\{0.20 + 0.20\left(\frac{2}{5}\right)^2 + 0.40\left(\frac{1}{5}\right)^2\right\}\frac{470^2}{1640}\right]^{1/2} = \pm 17 \text{ meters} \tag{7}$$

Of the total $D^2$, about 80% comes from the 0.20 part of the earth assumed to have a sounding-track spacing of 10°. The numerical values we have used may be too pessimistic, but in any case it seems clear that the dominant source of error is simply lack of data and that analyses using charts with scales of the order of 1:10,000,000 and contour intervals of the order of 500 to 1000 meters will suffice until most of the world is surveyed with spacing of 4° or better.

The most extensive previous analysis of the topography *published* is still that of *Prey* [1922; see also *Jung,* 1956]. In addition, there have been several unpublished analyses, of which probably the most detailed are those of G. J. Bruins [*Vening Meinesz,* 1962] and *Uotila* [1964], and there has been an analysis of the ocean function—1 over the oceans, 0 over the land—by *Munk and MacDonald* [1960].

If the analysis described herein is based on appreciably more detail than previous analyses, the effect should become apparent in an increase of the higher-degree rms variabilities $\sigma_n$ in Table 3. This increase is markedly true with respect to the analysis of *Prey* [1922] but not with respect to that of Bruins. Bruins did use older data—mainly the Monaco *Carte générale bathymétrique des océans,* 1:10,000,000, which in many areas have fewer soundings. He also estimated point values at about 1° intervals of arc, rather than area means, but it seems very unlikely that this would cause any perceptible aliasing at wave numbers of 35 or lower, considering the amount of variability within 1° and the great numbers of such points. Other indices of comparison used were

TABLE 5. Mean Values from Various Analyses of the Earth's Topography*

| | *Kossinna* [1933] | *Prey* [1921] | Bruins (Private Communication) | *Munk and MacDonald* [1960] | Paper (1966) |
|---|---|---|---|---|---|
| Oceanic area, % | 70.80 | (70.8) | (70.8) | 71.43 | 70.92 |
| Continental area, % | 29.20 | (29.2) | (29.2) | 28.57 | 29.08 |
| Mean world elevation, meters | −2430 | −2456 | −2367 | | −2300 |
| Mean land elevation, meters | 875 | 771 | 801 | | 756 |
| Mean ocean depth, meters | −3800 | −3787 | −3674 | | −3554 |

* Values in parentheses are assumed.

$$C_n = \sigma_n/(2n + 1)^{1/2} \tag{8}$$

$$\Delta C_n = \left[ \sum_{m=0}^{n} (\Delta A_n^{m^2} + \Delta B_n^{m^2})/(2n + 1) \right]^{1/2} \tag{9}$$

Where $\Delta A_n^m$, $\Delta B_n^m$ refer to the difference between two analyses. These values are given in Table 4 for $n \leq 8$. The $\Delta C_n$ seem unreasonably large relative to our estimate of the standard deviation $D$ in (7).

The discrepancies of this analysis from that of *Munk and MacDonald* [1960] are much larger, as indicated in Tables 2 and 4—considerably larger than can be attributed to their using point values at 5° intervals. For the one term for which other estimates are available, the zero order, our value agrees more closely, as shown by Table 5.

Although the restriction of this analysis to charts already published entails the omission of considerable recent bathymetry, the results indicate that an appreciable improvement in accuracy and detail of description of the topography over previously published analyses has been attained. Further details, including 5° × 5° mean values and comparisons with other analyses, are given by *Lee* [1966].

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